A Revisit to Voltage Partitioning Problem

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ABSTRACT

We revisit voltage partitioning problem when the mapped voltages of functional units are predetermined. If energy consumption is estimated by formulation $E = CV^2$, a published work claimed this problem was NP-hard. We clarify that it is polynomial solvable, then propose an optimal algorithm, its time complexity is $O\left(nk + k^2d\right)$ which is best so far, where n, k, and d are respectively the numbers of functional units, available supply voltages, and voltages employed in the final design. In reality, considering leakage power the energy-voltage curve is not simply monotonically increasing and there is still no optimal polynomial time algorithm. However, under the assumption that energy-voltage curve is quasiconvex, which is also a good approximation to actual situation, the optimal solution can be got in time $O\left(nk^2\right)$. Experimental results show that our algorithms are more efficient than previous works.

Categories and Subject Descriptors

B.7.2 [**Integrated Circuits**]: Design Aids

General Terms

Algorithm, Design

Keywords

Quasicovex Assumption,Voltage Partition

1. INTRODUCTION

Power consumption is a critical problem in modern VLSI design. With the increasing of power density, thermal issue has a great impact on the reliability of circuit. Multiple supplied voltages(MSV) design was suggested to trade off power and timing.

There are many previous works about the application of MSV. [2][8][9][12] integrated MSV to floorplanning and placement, [7][10][11][13] applied MSV in the post-floorplanning and post-placement stages, [3][5][6][14] considered MSV in high level synthesis. Gu *et al.*[14] used a novel slack distribution determine the mapped voltages of functional units under timing constrains, and then an optimal voltage partitioning algorithm for energy saving was proposed to generate voltage island. Liu *et al.*[6] dealt with the same problem as [14], he claimed that it was NP-hard, and gave a provably good approximation algorithm.

Considering leakage power, high-level supplied voltage does not mean high energy consumption. Sengupta *et al.*[5] extended the voltage partitioning problem for reality and proposed a greedy heuristic algorithm which is suboptimal. In this paper, we revisit this problem and summarize our contributions as follows.

- In the ideal situation that the energy consumption is estimated by the formulation $E = CV^2$, we prove that voltage partitioning problem for energy saving is not NP-hard, but polynomial solvable. We also propose an optimal algorithm which is more efficient than [6] and [14], see Table 1.
- In reality, energy consumption is not monotonically increasing with the increase of supplied voltage. Under the assumption that the energy-voltage curve is quasiconvex, we can get the optimal voltage partition in the running time that is comparable to [5], see Table 2.

The remainder of this paper is organized as follows. We review the voltage partitioning problem in section 2. Section 3 proves this problem can be solved in polynomial time in ideal situation. Our optimal algorithm is proposed in

Algorithm	Time Complexity	Space Complexity
[14]	$O\left(n^2d\right)$	$O\left(nd\right)$
[6]	$O\left(nkd\right)$	$O\left(n\right)$
Ours	$O\left(nk + k^2d\right)$	$O\left(n + kd\right)$

Table 1: The time and space complexity of algorithms when the energy is estimated by $E = CV^2$

Algorithm	Time Complexity	Space Complexity
[5]	$O\left(nkd^2\right)$	$O\left(nk\right)$
Ours	$O\left(nk^2\right)$	$O\left(nk\right)$

Table 2: The time and space complexity of algorithms for actual situation

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Table 3: Capacitances and Mapped Voltages Table

Function Unit	$u1$	$u2$	$u3$	$u4$	$u5$	$u6$
Capacitance(pF)	1.0	2.0	1.5	3.0	1.5	2.0
Mapped Voltage(V)	0.8	0.8	1.0	1.0	1.2	1.6

Section 4. Section 5 extends the problem when the energy-voltage curve is quasiconvex. Experimental results show in section 6. Finally, section 7 concludes this paper.

2. PROBLEM REVIEW

In the context, the mapped voltages of functional units have been predetermined according to [14], We define the voltage partitioning problem for energy saving as follows(we refer to some definition from [6]).

DEFINITION 2.1. *Given a set A of k available voltages,that $A = \{v_1^a, v_2^a, \cdots, v_k^a\}$, a set F of p function units, that $F = \{u_1, u_2, \cdots, u_p\}$. Each functional unit u_i has a mapped voltage $v_i (v_i \in A)$ and a capacitance c_i, and its energy consumption in supplied voltage $v_j^a (v_j^a \geq v_i)$ is $en_{ij} = c_i * \left(v_j^a\right)^2$. The energy of the set $en(F) = \sum_{i=1}^{p} c_i \times v^2(F)$, where $v(F) = \max_{i=1,\cdots,p} v_i$. And $s(i)$ is the final voltage that is assigned to u_i.*

PROBLEM 1. *(VPP) Given a positive integer d,a set A of k available voltages, that $A = \{v_1^a, v_2^a, \cdots, v_k^a\}$, and a set Γ of n functional units, that $\Gamma = \{u_1, u_2, \cdots, u_n\}$. Each functional unit u_i has a mapped voltage $v_i (v_i \in A)$ and a capacitance c_i. Find a voltage partition $\{F_1, F_2, \cdots, F_d\}$ such that $\cup_{i=1}^{d} F_i = \Gamma, F_i \cap_{\forall i \neq j} = \emptyset, F_i \neq \emptyset, \forall 1 \leq i, j \leq d$,and the total energy $\sum_{i=1}^{d} en(F_i)$ is the minimum.*

PROBLEM 2. *(OVPP) Given a positive integer d, a set A of k available voltages, that $A = \{v_1^a, v_2^a, \cdots, v_k^a\}$, and an ordered list L of n functional units, $L = \langle u_1, u_2, \cdots, u_n \rangle$.Each functional unit u_i has a mapped voltage $v_i (v_i \in A)$,$v_1 \leq v_2 \leq \cdots \leq v_n$, and a capacitance c_i. Find an ordered d-partition $\langle L_1, \cdots, L_d \rangle$, such that $L_i = \langle u_p, u_{p+1}, \cdots, u_q \rangle, |L_i| \neq 0, p = q - |L_i| + 1, q = \sum_{j=1}^{i} |L_j|$, and the total energy $\sum_{i=1}^{d} en(L_i)$ is the minimum.*

See the example shown in Table 3. If we partition the functional units set $\Gamma = \{u_1, u_2, \cdots, u_6\}$ into 3 disjoint subsets, $F_1 = \{u_1, u_2, u_3, u_4\}, F_2 = \{u_5\}, F_3 = \{u_6\}$, then $en(F_1) = (1.0 + 2.0 + 1.5 + 3.0) \times 1.0^2 = 7.5$, $en(F_2) = 1.5 \times 1.2^2 = 2.16$, $en(F_3) = 2.0 \times 1.6^2 = 8.96$, the total energy is $\sum_{i=1}^{3} en(F_i) = 14.78$.

3. POLYNOMIAL SOLVABLE

Liu *et al.*[6] claimed that VPP was NP-hard and the optimal solution of OVPP was at most α^2 times the best solution of VPP, where α is the constant ratio of the maximum to the minimum available voltages. In this section, we prove that OVPP is not approximate to VPP, in fact they are equal, if the energy-voltage curve is monotonically increasing.

DEFINITION 3.1. *In a partition set $F_q (1 \leq q \leq d)$ for VPP, Let $v_{max} = \max_{\{i|u_i \in F_q\}} v_i$, $v_{min} = \min_{\{i|u_i \in F_q\}} v_i$. $\forall u_i \in \Gamma, if\ v_{min} \leq v_i \leq v_{max}, then\ u_i \in F_q$. We say that F_q is **continuous**.*

LEMMA 3.1. *In the optimal solution for VPP, $\forall u_i, u_j \in \Gamma, if\ v_i = v_j, then\ s(i) = s(j)$.*

PROOF. Proof by contradiction. Suppose $s(i) \neq s(j)$, without loss of generality, let $s(i) < s(j)$. So we can get a better solution if we assign $s(i)$ to u_j. □

LEMMA 3.2. *We can get a optimal solution for VPP, that $\forall u_i, u_j \in \Gamma$, if $v_i = v_j$, then they are partitioned to the same set.*

PROOF. Suppose in the optimal solution for VPP, there are two functional units u_i and u_j, that satisfies $v_i = v_j$, but in the different partition sets F_p and F_q. According to *lemma* 3.1, $s(i) = s(j)$, so we can merge F_p and F_q without loss of optimality. □

LEMMA 3.3. *There is an optimal solution for VPP, that each partition set F is **continuous**.*

PROOF. Suppose a partition set F_q is not continuous in the optimal solution for VPP, let $v_{max} = \max_{\{i|u_i \in F_q\}} v_i$, $v_{min} = \min_{\{i|u_i \in F_q\}} v_i$. So there is some functional unit, $u_i (\ v_{min} \leq vi \leq v_{max})$, that is not belong to F_q.

We show that $s(i)$ is impossible to be less than v_{max}, because we can assign $s(i)$ to the function units, whose mapped voltages are between v_{min} and $s(i)$ in F_q , to get a better solution.

We also show that $s(i)$ is impossible to be greater than v_{max}, because we can assign v_{max} to u_i to get a better solution.

To sum up, $s(i) = v_{max}$. Including *lemma* 3.2, we establish this lemma. □

THEOREM 1. *VPP is equal to OVPP,and it is not NP-hard.*

PROOF. VPP is equal to OVPP according to *lemma* 3.3. Since OVPP is polynomial solvable, VPP is not NP-hard iff $P \neq NP$. □

4. OPTIMAL ALGORITHMS

We first review the previous optimal algorithms for OVPP. Gu *et al.*[14] used the dynamic programming, because OVPP has the optimal subtraction. Liu *et al.*[6] took note of that the functional units, which had the same mapped voltage, were not necessary to be divided to different partition sets in the optimal solution, then speeded up the dynamic programming. The time complexity of [14] and [6] are respectively $O\left(n^2 d\right)$ and $O(nkd)$.

input : n functional units, k available voltages in technology library, partition number d
output: optimal voltage partition for energy saving
Construct a $DAG = (V, E)$, $V = \{i | v_i^a \in A\} \cup \{0\}$, $E = \{e_{ij} | j > i\}$;
Compute the cost of each edge in the DAG;
Find the shortest max-d-stop path in the DAG;

Algorithm 1: Optimal Algorithm for VPP

Now we propose an optimal algorithm for VPP. The framework of our algorithm is shown in Algorithm 1. Let U_i denote the set in which each functional unit has the same

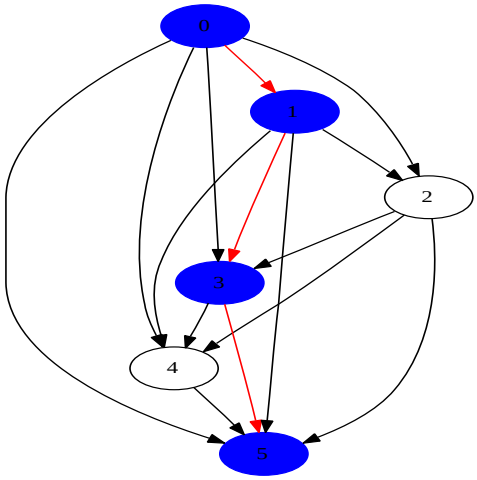

Figure 1: The DAG in ideal situation when $k = 5, d = 3$. The edges and nodes of shortest d-stop path are respectively colored red and green.

mapped voltage v_i^a, P_{ij} denote the total energy consumption of U_i in the supply voltage $v_j^a (v_j^a \geq v_i^a)$. We construct a $DAG = (V, E), V = \{i | v_i^a \in A\} \cup \{0\}, E = \{e_{ij} | j > i\}$. Each edge e_{ij} has a cost, that can be presented as $cost\,(e_{ij}) = \sum_{i+1 \leq z \leq j} P_{zj}$, which is also the part of energy consumption when choosing v_i^a and v_j^a as supplied voltage, but not v_{i+1}^a, $\ldots$, v_{j-1}^a.

DEFINITION 4.1. *A path $\rho = (0, n1, n2, \cdots, k)$ is called a **max-d-stop path** if it has at most **d** nodes excluding the start node 0. If it has exactly **d** nodes excluding 0, we call it **d-stop path**.*

THEOREM 2. *A **d-stop path** and a **feasible solution** of OVPP is one-to-one mapping, and the **shortest d-stop path** is corresponding to **the optimal solution** of OVPP.*

PROOF. Let $n_0 = 0$ and $v_0^a = 0$. For any d-stop path $\rho = (0, n1, n2, \cdots, k)$, we can divide functional units set $\Gamma = \{u_1, u_2, \cdots, u_n\}$ into d disjoint sets $F_1, F_2, \cdots, F_d$, that $F_i = \left\{u_i | u_i \in \Gamma \cap v_{n_{i-1}}^a < v_i \leq v_{n_i}^a\right\}$, vice versa. It is apparently that $cost\left(e_{n_{i-1}n_i}\right) = en\,(F_i)$, so the cost of ρ is equal to $\sum_{i=1}^{d} en\,(F_i)$. The *theorem* is established. □

Fig. 1 shows an example. Suppose the path $\rho = (0, 1, 3, 5)$ is the shortest d-stop path in DAG, the corresponding partition sets are: $F_1 = \{u_i | v_i \leq v_1^a\}, F_2 = \{u_i | v_1^a < v_i \leq v_3^a\}$, $F_3 = \{u_i | v_3^a < v_i \leq v_5^a\}$. The total energy consumption is $\sum_{e \in \rho} e = e_{01} + e_{13} + e_{35}$.

We analyze the time complexity of our algorithm. The construction of DAG need $O\,(nk)$. The shortest d-stop path in DAG can be found in $O\,(|E|d)$, where $|E| = O\left(k^2\right)$. So the total running time is $O\left(nk + k^2 d\right)$.

5. QUASICONVEX ASSUMPTION

Since the actual energy-voltage curve may not be monotonically increasing, OVPP is no more equal to VPP. Sengupta *et al.*[5] proposed a greedy heuristic algorithm whose time complexity is $O\left(nkd^2\right)$, but there is still no optimal algorithm in the polynomial running time. However, under the assumption that the energy-voltage curve is quasiconvex, it is polynomial solvable using the similar method.

We construct the same $DAG = (V, E)$ as section 4, but the cost of edge is computed in the different way. For any functional unit u_m, We mark o_m as the voltage in which its energy consumption is minimum, the edge set E can be divided into three disjoint parts $\Omega_{1m}, \Omega_{2m}, \Omega_{3m}$. Let v_0^a=0, the three disjoint parts can be represented as follows:

$$\Omega_{1m} = \left\{e_{ij} | v_i^a < v_m \cap v_j^a \geq v_m\right\} \tag{4}$$

$$\Omega_{2m} = \left\{e_{ij} | v_m \leq v_i^a < o_m \cap v_j^a > v_i^a\right\} \tag{5}$$

$$\Omega_{3m} = E - \Omega_{1m} - \Omega_{2m} \tag{6}$$

The contribution to each edge $contri_{u_m}\,(e)$ is shown below:

$$\begin{cases} c_m \times \left(v_j^a\right)^2 & \text{if} \quad e_{ij} \in \Omega_{1m}; \\ \min\left\{0, c_m \times \left[\left(v_j^a\right)^2 - \left(v_i^a\right)^2\right]\right\} & \text{if} \quad e_{ij} \in \Omega_{2m}; \\ 0 & \text{if} \quad e_{ij} \in \Omega_{3m}. \end{cases} \tag{7}$$

Fig. 2 depicts the three edge sets vividly, Suppose a function unit u_m, its mapped voltage $v_m = v_2^a$(the *red* vertex), and $o_m = v_3^a$(the *yellow* vertex), so the three disjoint sets are Ω_{1m}(the *blue* edges),Ω_{2m}(the *green* edges), and Ω_{3m}(the *black* edges). Next,we prove its correctness.

LEMMA 5.1. *For any path ρ from 0 to k in $DAG = (E, V)$, $\forall u_m \in \Gamma$, $\sum_{\{e | e \in E \cap e \in \rho\}} contri_{u_m}\,(e) = \min_{\{i | i \in V \cap i \in \rho\}} en_{mi}$.*

PROOF. Let $\rho 1$ denote the section of ρ, that each node i of $\rho 1$ satisfies $v_m \leq v_i^a \leq o_m$, $\rho 2$ denote the section of ρ, that each node i of $\rho 2$ satisfies $o_m < v_i^a \leq v_k^a$. Apparently $\sum_{\{e | e \in E \cap e \in \rho\}} contri_{u_m}\,(e) = \sum_{\{e | e \in \rho 1\}} e + e_{lf} = \min_{\{i | i \in V \cap i \in \rho\}} en_{mi}$, where l is the last node of $\rho 1$, f is the first node of $\rho 2$. □

THEOREM 3. *Under the assumption that the energy-voltage curve is quasiconvex, the **shortest max-d-stop path** in the DAG is corresponding to **the optimal solution** of VPP.*

PROOF. For any path ρ from 0 to k in $DAG = (V, E)$, $\sum_{\{e | e \in \rho\}} cost\,(e) = \sum_{i=1}^{n} \sum_{\{e | e \in \rho\}} contri_{u_i}\,(e)$, which is also equal to $\sum_{i=1}^{n} \min_{\{j | j \in V \cap j \in \rho\}} en_{ij}$ according to *lemma* 5.1. And the partition number is not allowed to exceed d, so the *theorem* is established. □

Time Complexity

The construction of DAG need $O\left(nk^2\right)$, plus the time of finding shortest max-d-stop path, which is also $O\left(k^2 d\right)$, the total time complexity is $O\left(nk^2\right)$.

6. EXPERIMENTAL RESULTS

We implemented our algorithms in C++ on a PC(3.0GHz CPU, 1GB Memory) running linux OS. We also implemented Liu *et al.*'s algorithm for comparison. Same as [6], the five available voltages in technology library are: 0.8, 1.0, 1.2, 1.4, 1.6(V), the capacitance is randomly generated with means 20 pF by [4]. The only difference is that, for every functional unit the mapped voltage is among the k available voltages with the same possibility in our experiment, because how to compute its value is another problem that will not affect the

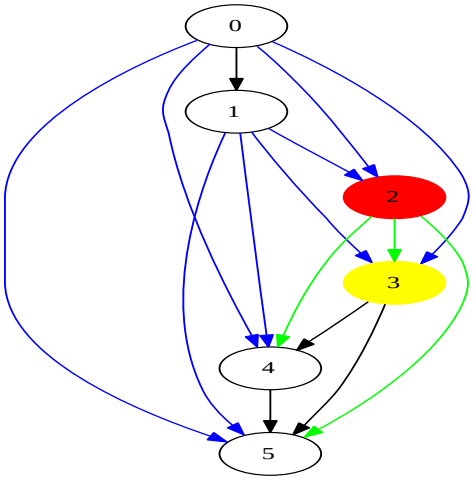

Figure 2: The $DAG = (V, E)$ when $k = 5, d = 3$ and the energy-curve is convex in the discrete available voltages.

correctness and efficiency of algorithms. To verify the optimality, we also formulate the voltage partitioning problem as an integer linear programming(ILP) and solve it by open source ILP solver *CBC*[1].

Table 4 reports experimental results. We can see that, Liu *et al.*'s algorithm and ours, both achieved the optimal power saving, and our algorithm is at least twice as fast as Liu *et al.*'s. It can be predictable that our algorithm is much more efficient than Liu *et al.*'s due to its low time complexity.

For reality, when k and d is small(for example $k = 5, d = 3$), Liu *et al.*'s algorithm and the greedy heuristic algorithm works well. However, with the increasing of k and d, both of them may be slow and get into sub-optimal solution. But our algorithm is still efficient and exact.

7. CONCLUSIONS

We have proved that the voltage partitioning problem for energy saving is polynomial solvable when the mapped voltages of functional units are predetermined and the energy consumption is estimated by $E = CV^2$, then proposed a very efficient algorithm to solve it. We also extended this problem for reality. Under the assumption that the energy-voltage curve is quasiconvex, our algorithm is still very efficient and keep optimal.

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Table 4: Voltage Partitioning Results In Ideal Situation(k=5,d=3)

	# of Units	Power Consumption(%)		Running Time(s)	
		[6]	Ours	[6]	Ours
tp1	$1E5$	65.91	65.91	0.01	< 0.01
tp2	$2E5$	65.94	65.94	0.02	0.01
tp3	$1E6$	66.23	66.23	1.12	0.45
tp4	$2E6$	66.21	66.21	2.23	0.89
tp5	$1E7$	66.21	66.21	11.4	4.6
Average		66.23	66.23	2.48×	1