Efficient Power Network Analysis with Complete Inductive Modeling

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Abstract
In this paper, an efficient method is proposed to accurately analyze large-scale power/ground (P/G) networks, where inductive parasitics are modeled with the partial reluctance. The method is based on frequency-domain circuit analysis and the technique of vector fitting [8], and obtains the time-domain voltage response at given P/G nodes. The frequency-domain circuit equation including partial reluctances is derived, and then solved with the GMRES algorithm with rescaling and precondition techniques. Due to the sparsified reluctance matrix and the frequency-domain based simulation techniques, the proposed method is able to handle large-scale P/G networks with complete inductive modeling. Numerical results show that the proposed method is orders of magnitude faster than HSPICE, and capable of handling the inductive P/G structures with more than 100,000 wire segments.

Keywords
Power network, inductive effect, reluctance, frequency-domain

1. Introduction
As VLSI circuit integrates more than thousand million transistors with working frequency of multiple giga-hertz (GHz), the circuit’s power consumption increases exponentially. This calls for low-power design techniques, and increases the request of performing accurate and complete full-chip analysis to guarantee power integrity. Therefore, accurate modeling and dynamic simulation of the power/ground (P/G) grid is becoming critical for VLSI circuit design and verification.

Modeling the inductive effect of on-chip and off-chip interconnects is another research focus for current nano-scale VLSI chip. The conventional RC model of interconnect is not good enough for accurate circuit analysis. Moreover, the reduction of resistance by copper and capacitance by low-k dielectric highlights the inductive effect; the denser geometries and growing complexity of interconnect structures bring challenges to the power grid simulation with inductance modeling.

In the past few years, the main focus of power network analysis has been on how to determine the trade-off between the simulation accuracy and the speed. Many previous works focused on the efficient time-domain transient analysis of large-scale power networks. In some works, the circuit simulation is accelerated by fast linear equation solvers. They include the direct solver “KLU” [1], the generalized minimal residual (GMRES) method [2] and iterative solvers implemented with the preconditioned conjugate gradient (PCG) method [3]. In others, the circuit size is reduced by using methods such as circuit partitioning [4] and hierarchical model reduction [5].

All the works above focus on the efficient time-domain transient analysis of power networks. A frequency-domain based simulation method was proposed to obtain the time-domain voltage response in [6, 7]. By the vector fitting technique [8], the frequency-domain voltage responses are transformed into time-domain voltage response. However, the existing work [6] does not consider the inductive coupling of P/G wires.

In this paper, an efficient frequency-domain based simulation method for P/G grid with complete inductance model is proposed. With modified nodal analysis (MNA) [9], we firstly derive the frequency-domain circuit equation including parasitic inductances, and then replace the inductance matrix with sparsified reluctance matrix. The GMRES algorithm with rescaling and precondition techniques is proposed to solve the reluctance based circuit equation. Numerical results show the proposed method gains tens to hundreds speedup over the HSPICE with preserving high accuracy. It is also demonstrated that the proposed method could handle large inductive P/G structures with more than 100,000 wire segments.

This paper is organized as follows: Section 2 introduces the background of the power network model and the frequency-domain based simulation method. Section 3 introduces the fast power network analysis considering the reluctance parameters. Section 4 gives the numerical results. And finally there is the conclusion.

2. Background
2.1 Power Network with Complete Inductive Modeling
On-chip power network is usually routed in several metal layers to form mesh structure. In each layer, the orientation of metal wires is along either X-axis or Y-axis, alternatively. And, the power wires are interlaced with the ground wires. Between two adjacent layers, the P/G wires are connected through vias, which cut the wires into small wire segments. Fig. 1 shows the 3-D view of a portion of two-layer power network. For complete electromagnetic modeling of power network, the partial element equivalent circuit (PEEC) technique should be employed, which results in a circuit
including resistance, capacitance and inductance elements for each wire segment. Time-varying current sources are connected to some circuit nodes, characterizing the supply current for active circuit modules. These current sources draw current from the power network and cause voltage fluctuations. The waveform of current source is usually described as a piecewise linear (PWL) function.

In the PEEC model, the inductive effect is characterized with the partial inductance, including self and mutual inductances. For the P/G grid with a large amount of wire segments, the partial inductance matrix in circuit equation will be a large-scale dense matrix and is hard to be sparsified while preserving accuracy. This makes it prohibitive to extract and simulate the P/G grid with the partial inductance. A concept of partial reluctance (or, $K$-element) was recently proposed; the partial reluctance matrix $K$ is defined to be the inverse of partial inductance matrix $L$:

$$K = L^{-1}. \quad (1)$$

Related works showed that the partial reluctance has the locality property like capacitance so that it could be easily sparsified. With the sparsified partial reluctance matrix, the circuit simulation is not only largely accelerated but also stable [10, 11]. Efficient techniques were also proposed to extract the partial reluctance for large interconnect structures [11-13].

As the clock frequency continues to increase, the high-frequency effect should be considered for wider global interconnects, such as the high-level P/G wires. The reluctance extraction techniques considering high-frequency effect were proposed in [12, 13]. In this work, they are utilized to extract the frequency-dependent reluctance and resistance parameters for upper-layer P/G wires.

### 2.2 Time-Domain Simulation Based on Frequency-Domain Analysis and Vector Fitting

Fig. 2 describes the flow of the frequency-domain based simulation method [6]. The time-domain waveform of current sources is firstly converted to frequency-domain expression with Laplace transformation. Since each input current source $I(t)$ is described as a PWL function, its frequency-domain expression can be derived analytically. Then, the circuit equation for frequency-domain analysis can be formulated, and solved for each specified frequency. With suitable frequency sampling, the vector fitting technique [8] is adopted to fit the frequency-domain voltage responses $V(s)$ with a partial fractional expression $\tilde{V}(s)$. Finally, the result of vector fitting is converted to the time-domain voltage waveform $V(t)$ [6, 7]. For power network analysis, the voltage responses on a small amount of nodes are usually needed. So, this frequency-domain based simulation method has large advantages over the conventional time-domain transient simulation. Previous works show it is orders of magnitude faster than the conventional time-domain simulation methods, while preserving sufficient accuracy [6, 7].

In [6], only the nodal analysis (NA) technique was used so that it could not deal with the complete P/G parasitic circuit with inductive elements. The solution technique for the frequency-domain circuit equation in [6] could also be improved. In this paper, we extend the work in [6] with the modified nodal analysis (MNA) to handle the P/G grid with reluctance elements. And, the GMRES method [14] with rescaling and preconditioning techniques is developed to efficiently solve the frequency-domain circuit equation.

### 3. Fast Power Network Analysis Considering Partial Reluctances

#### 3.1 Basic Idea

To apply the frequency-domain based simulation method for the P/G grid modeled with partial reluctances, we need to derive the frequency-domain circuit equation using the MNA technique. Firstly, we derive the equation with inductance elements. Then, the inductance matrix is replaced with the reluctance matrix through (1). The sparsified reluctance matrix extracted with the DRRE method [12, 13] is used here.

Because the capacitance and reluctance matrices in the circuit equation could be very sparse, the iterative GMRES algorithm is considered to solve the circuit equation at each frequency point. Along with efficient technique of storing sparse matrix, the solver will be very efficient for the frequency-domain analysis of large-scale P/G grid. For the selection of frequency samples, we follow the technique in [6]. The highest frequency of the voltage response $f_{max}$ is usually no larger than several tens of GHz, and the logarithmic scale sampling of frequency is adopted. So, the number of frequency points is of $O(\log f_{max})$, and is about several tens in our experiments.

![Figure 1. A two-layer structure of P/G grid](image)

![Figure 2. The flow chart of the frequency-domain based simulation method](image)
different frequency, the SPICE input file may be different. This does not bring difficult to the frequency-domain based simulation method, while the conventional time-domain simulation is not able to handle these frequency-dependent parameters.

3.2 Frequency-Domain Circuit Equations

Fig. 3 shows the PEEC model for a small portion of the P/G grid, which includes four wire segments. The mutual inductions between wire segments and coupling capacitance between nodes are considered, but for clear view they are not drawn in Fig. 3. If setting the variables of nodal voltage and branch current as shown in Fig. 3, the circuit equation can be established with the MNA technique:

$$
\begin{bmatrix}
G & A_L^T & V_n(t) \\
-A_L & R & I_L(t)
\end{bmatrix} + \begin{bmatrix}
C & 0 & dV_n(t) \\
0 & L & dI_L(t)
\end{bmatrix} = \begin{bmatrix}
-A_L^T I_n(t) \\
0
\end{bmatrix},
$$

(2)

where $I_n(t)$, $V_n(t)$ and $I_L(t)$ are the vectors of independent current sources, the unknown nodal voltages and the unknown currents through inductor branches, respectively. The $G$ matrix includes the conductances of resistors not in the inductor branches, while $R$ includes the resistances on the inductor branches. $C$ and $L$ are the capacitance and inductance matrices, respectively. Matrix $A_L$ and $A_L$ are the adjacent matrix for the current source and inductors, respectively.

For frequency-domain analysis, Eq. (2) is converted to:

$$
\begin{bmatrix}
G & A_L^T & V_n(t) \\
-A_L & R & I_L(t)
\end{bmatrix} + s \begin{bmatrix}
C & 0 & V_n \\
0 & L & I_L
\end{bmatrix} = \begin{bmatrix}
-A_L^T I_n(t) \\
0
\end{bmatrix},
$$

(3)

where $s = j\omega$ and $\omega$ is the angular frequency. Here $I_n$ stands for the frequency-domain expression of current sources, which is obtained with the Laplace transform of time-domain waveforms. For a given frequency, the frequency-domain voltage response can be obtained by solving the complex-valued linear equation system (3). The number of unknowns in (3) equals to the number of circuit nodes $n_v$ plus the number of inductor branches $n_L$.

The complex-valued $I_n$, $V_n$ and $I_L$ in (3) can be decomposed into real and imaginary parts:

$$I_n = I_{re} + jI_{im},
$$

$$V_n = V_{re} + jV_{im},
$$

$$I_L = I_{re} + jI_{im}.
$$

Then, (3) can be transformed into a real-valued linear equation system:

$$
\begin{bmatrix}
G - sC & A_L^T & 0 \\
-sC & G & 0 \\
-A_L & 0 & R
\end{bmatrix} \begin{bmatrix}
V_{re} \\
V_{im} \\
I_{re} + jI_{im}
\end{bmatrix} = \begin{bmatrix}
-A_L^T I_{re} \\
-A_L^T I_{im} \\
0
\end{bmatrix}.
$$

(4)

We replace the $L$ matrix in (4) with $K^{-1}$, where $K$ is the partial reluctance matrix, and then derive the circuit equation with reluctance elements:

$$
\begin{bmatrix}
G - sC & A_L^T & 0 \\
-sC & G & 0 \\
-A_L & 0 & R
\end{bmatrix} \begin{bmatrix}
V_{re} \\
V_{im} \\
I_{re} + jI_{im}
\end{bmatrix} = \begin{bmatrix}
-A_L^T I_{re} \\
-A_L^T I_{im} \\
0
\end{bmatrix}.
$$

(5)

where 1 stands for the identify matrix. After solving (5), we can get the frequency-domain voltage responses at the specified frequency point.

3.3 Efficient Technique to Solve the Frequency-Domain Equations

The dimension of the linear equation system (5) could be very huge for a large-scale power network, but the coefficient matrix may be very sparse. Efficient storing and solving techniques for sparse matrix are required to perform the frequency-domain analysis.

A length-varied 2-D array scheme is utilized to store the non-zero entries in the coefficient matrix of (5). This storing scheme was proposed in [15], and can be regarded as a variation of the block compressed row storage (BCRS) scheme. The work on 3-D capacitance extraction shows that this matrix storage technique is very efficient and suitable for iterative equation solvers like GMRES [15]. The multiplications of $KA_L$ and $KR$ are performed prior to the equation solution procedure. Due to the property of matrix $A_L$, $KA_L$ is still a sparse matrix, with a little more non-zero entries than $K$. On the other hand, since $R$ is a diagonal matrix, $KR$ becomes a sparse matrix with same non-zero pattern as $K$.

The rescaling technique is used to balance the order of magnitude of the coefficients in (5). We find out that the non-zero entries in $KR$ have much less value than that in matrix $G$. So, a rescaling factor $\delta$ is multiplied to the last two block rows of (5), resulting in

$$
\begin{bmatrix}
G - sC & A_L^T & 0 \\
-sC & G & 0 \\
-A_L & 0 & R
\end{bmatrix} \begin{bmatrix}
V_{re} \\
V_{im} \\
I_{re} + jI_{im}
\end{bmatrix} = \begin{bmatrix}
-A_L^T I_{re} \\
-A_L^T I_{im} \\
-KA_L I_{re}
\end{bmatrix}.
$$

(6)

The value of $\delta$ is selected to be the ratio of a typical non-zero entry of $G$ to a typical non-zero entry of $KR$.

After rescaling, the condition number of the coefficient matrix is largely reduced, and the convergence rate of GMRES algorithm is also improved. In Table 1, the condition numbers for the coefficient matrices before and after the rescaling are listed. They correspond to two examples of P/G grid and two different frequencies. From the
The upper bound of frequency, the time complexity of an equation dominates the total computational time, because the approximation.

The Jacobi preconditioner could remarkably improve the convergence rate of the GMRES algorithm with almost no extra computational cost [15]. However, since there may be zero diagonal items in matrix $G$, the Jacobi preconditioner cannot be used directly. At the position where the diagonal item of $G$ is zero, we assign one to the corresponding item of preconditioner. Since the number of zero diagonals in $G$ matrix is much smaller than the total dimension of the coefficient matrix, the constructed preconditioner approximates to the Jacobi preconditioner very well. Our experiments show that the solution of (6) is largely accelerated with this precondition technique.

### 3.4 Algorithm Flow and Discussion

The proposed algorithm can be summarized as follow:

1. Convert the current sources from time-domain waveform to frequency-domain expression with Laplace transform.
2. Form the frequency-domain equations (6) with extracted reluctance, resistance and capacitance parameters.
3. Solve the equations (6) using GMRES method with rescaling and precondition techniques to get the frequency-domain voltage responses.
4. Repeat the steps 2 and 3 for each frequency points to get the voltage responses for all the frequency-domain voltage responses.
5. Obtain the time-domain voltage response using the vector fitting method.

Because the partial reluctance matrix is very sparse, and so is the matrix $A_L$, the multiplication process of $K$ and $A_L$ is very fast. And the result matrix $KA_L$ is still a sparse matrix with few fill-ins. Because matrix $R$ is a diagonal matrix, the multiplication process is fast and the result matrix $KR$ is as sparse as matrix $K$.

For equations solution, we use GMRES method with rescaling and precondition technique. And with the compressed row storage method, the complexity of solving one equation is $O(N^{\alpha})$, where $N$ is the dimension of (6), and $\alpha$ is a quantity between 1 and 2.

We adopt the logarithmic scale sampling of frequency. Then, the number of frequency points is $O(\log f_{\text{max}})$, where $f_{\text{max}}$ is the upper bound of frequency. The time complexity of vector fitting is $O(N^2 \cdot \log f_{\text{max}})$, where $N_0$ is the order of approximation.

For large-scale power network, the time for solving equation dominates the total computational time, because the dimension of (6) $N$ is much larger than $N_0$. So the time complexity for solving the frequency-domain linear equation system is about $O(N^\alpha \cdot \log f_{\text{max}})$. If the voltage responses of multiple nodes on power network are considered, the time for vector fitting will be multiplied by the number of output nodes $N_{\text{out}}$. For analysis of maximum voltage variation, only the nodes at the lowest level of P/G grid are considered. Therefore, $N_{\text{out}}$ is a small number.

From above analysis, we know that the frequency-domain based simulation method with mutual reluctance parameters can save memory and computation time than that based on mutual inductance. And it is proportional to the number of frequency samples, and is not related with the number of time steps as in a conventional time-domain transient simulation method. So it is faster than time-domain transient simulation method.

### 4. Numerical Results

The proposed simulation method is implemented in C language. A Matlab program is written to take in the frequency-domain responses and convert them to the time-domain voltage waveform with the help of vector fitting. We compare the proposed simulation method with commercial simulators HSPICE. All experiments are run on a server using Intel Xeon CPU of 3GHz with 4GB memory.

For mesh structured power network with complete inductive model using partial reluctance, we get the frequency-domain responses at 38 frequency points from DC to 3.5 GHz. Fig. 4 shows the result of the frequency-domain voltage responses and its fitting result with vector fitting technique. In Fig. 5, the time-domain voltage waveform converted from the partial fractional expression is compared with that obtained from transient simulation of HSPICE. From Fig. 5, we see that the voltage waveforms obtained with both method have little discrepancy. The relative error is only 1.9% for the minimum voltage and 1.0% for the maximum voltage.

For five test cases of P/G network, the simulation times of the proposed method and HSPICE are listed in Table 2. The time for proposed method just includes that for establishing the frequency-domain responses and converging to time-domain voltage waveform, the total CPU time of the proposed method would be a little more than that in Table 2. The fifth column gives the time for the GMRES method with both rescaling and precondition technique, from which we could clearly see the efficiency benefit brought by the preconditioning. For the larger two cases, the GMRES can not converge without preconditioning. The last column is the speed up of the proposed method with respected to HSPICE. We get the partial reluctance by DRE method [12, 13], and inverse it to get the partial inductance as the input of HSPICE. For the third case, HSPICE could not get the result for out of memory, and for the last two cases, because the dimension of the partial reluctance matrix is too large, we can not get its inverse matrix. The proposed method gains tens to hundreds speedup over HSPICE for the first two cases. The proposed method is also able to handle large P/G structures that HSPICE can not afford, as shown in Table 2.

### Table 1. The comparison of condition number

<table>
<thead>
<tr>
<th>Examples</th>
<th>Frequency</th>
<th>Before rescaling</th>
<th>After rescaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>100Hz</td>
<td>1.04×10¹⁴</td>
<td>1.72×10¹⁰</td>
</tr>
<tr>
<td></td>
<td>3.5Ghz</td>
<td>1.48×10¹³</td>
<td>3.82×10¹⁰</td>
</tr>
<tr>
<td>Example 2</td>
<td>100Hz</td>
<td>1.77×10¹¹</td>
<td>3.33×10¹⁰</td>
</tr>
<tr>
<td></td>
<td>3.5Ghz</td>
<td>2.69×10¹⁰</td>
<td>8.31×10⁹</td>
</tr>
</tbody>
</table>

Examples Frequency Before After

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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### Table 2. The comparison of simulation times

<table>
<thead>
<tr>
<th>Examples</th>
<th>Frequency</th>
<th>Proposed Method</th>
<th>HSPICE</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From above analysis, we know that the frequency-domain based simulation method with mutual reluctance parameters can save memory and computation time than that based on mutual inductance. And it is proportional to the number of frequency samples, and is not related with the number of time steps as in a conventional time-domain transient simulation method. So it is faster than time-domain transient simulation method.

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From Table 2, we see that the proposed method is orders of magnitude faster than HSPICE. The proposed method has the potential of parallelizing the solution of frequency-domain equation (6) for the frequency samples. If parallel programming is applied, there will be potentially a larger speed up over HSPICE.

<table>
<thead>
<tr>
<th>Examples</th>
<th>#nodes</th>
<th>#segments</th>
<th>Time of proposed method (rescaling)</th>
<th>Time of proposed method (rescaling &amp; precondition)</th>
<th>Time of HSPICE</th>
<th>Speedup ratio to HSPICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>348</td>
<td>344</td>
<td>31.3</td>
<td>19.5</td>
<td>381.9</td>
<td>19.6</td>
</tr>
<tr>
<td>Example 2</td>
<td>808</td>
<td>804</td>
<td>107.6</td>
<td>73.9</td>
<td>9693.7</td>
<td>131</td>
</tr>
<tr>
<td>Example 3</td>
<td>1460</td>
<td>1468</td>
<td>307.9</td>
<td>209.7</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Example 4</td>
<td>10832</td>
<td>10828</td>
<td>N.A.</td>
<td>7383.5</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Example 5</td>
<td>103344</td>
<td>103340</td>
<td>N.A.</td>
<td>354406</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

Figure 4. A frequency-domain response and its fitting result with the vector fitting technique

Figure 5. Comparison of the voltage responses obtained by HSPICE and the proposed method

5. Conclusions

An efficient framework is proposed for the P/G network analysis with complete inductive model which includes two main contributions:

(1) A frequency-domain based simulation method is developed to take advantage of the sparsified reluctance matrix. And, the method can collaborate with the frequency-dependent parameters to model the high-frequency effect.

(2) The techniques of storing sparse matrices, rescaling and preconditioning are proposed to enable fast GMRES solution of the frequency-domain circuit equation. Numerical results show that the proposed simulation method has a large advantage over the conventional time-domain simulation tool HSPICE, and is able to simulate the complete parasitic effects for large-scale P/G structures.

6. References


